

Tutorial: 05

Subject Code: 3110015

Year: 2018 - 2019

Line Integral

1. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to point $(1, 1, 1)$.
2. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segments $C_1 : \bar{r}(t) = t\hat{i} + t\hat{j}, \quad 0 \leq t \leq 1$ and $C_2 : \bar{r}(t) = \hat{i} + \hat{j} + t\hat{k}, \quad 0 \leq t \leq 1$ joining the origin to point $(1, 1, 1)$.
3. Evaluate $\int_C (x + y) ds$ where C is the straight line segment $x = t, y = (1 - t), z = 0$, from $(0, 1, 0)$ to $(1, 0, 0)$.
4. Find the line integral of $f(x, y, z) = x + y + z$ over the straight line segment from $(1, 2, 3)$ to $(0, -1, 1)$.
5. Integrate $f(x, y, z) = x + \sqrt{y} - z^2$ over the path from $(0, 0, 0)$ to $(1, 1, 1)$ where,

$$C_1 : \quad \bar{r}(t) = t\hat{i} + t^2\hat{j}, \quad 0 \leq t \leq 1$$

$$C_2 : \quad \bar{r}(t) = \hat{i} + \hat{j} + t\hat{k}, \quad 0 \leq t \leq 1$$

6. Evaluate $\int_C x ds$, where C is

(1) the straight line segment $x = t, y = t/2$, from $(0, 0)$ to $(4, 2)$

(2) the parabolic curve $x = t, y = t^2$, from $(0, 0)$ to $(2, 4)$

7. Find the line integral of $f(x, y) = ye^{x^2}$ along the curve $\bar{r}(t) = (4t)\hat{i} - (3t)\hat{j}, \quad -1 \leq t \leq 2$.
8. Evaluate $\int_C \frac{x^2}{y^{4/3}} ds$, where C is the curve $x = t^2, y = t^3$ for $1 \leq t \leq 2$.
9. Find the work done by the force field $\bar{F} = (y - x^2)\hat{i} + (z - y^2)\hat{j} + (x - z^2)\hat{k}$ in moving an object along the curve $\bar{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}, \quad 0 \leq t \leq 1$, from $(0, 0, 0)$ to $(1, 1, 1)$.

10. Find the circulation of the field $\vec{F} = (x - y)\hat{i} + x\hat{j}$ around the circle $\vec{r}(t) = (\cos t)\hat{i} + (\sin t)\hat{j}$, $0 \leq t \leq 2\pi$.
11. Find the flux of the field $\vec{F} = (x - y)\hat{i} + x\hat{j}$ around the circle $\vec{r}(t) = (\cos t)\hat{i} + (\sin t)\hat{j}$, $0 \leq t \leq 2\pi$.
12. Find the work done by the conservative field $\vec{F} = (yz)\hat{i} + (xz)\hat{j} + (xy)\hat{k}$ where $f(x, y, z) = xyz$ is moving an object along any smooth curve C joining the point $A(-1, 3, 9)$ to $B(1, 6, -4)$.
13. Show that $\vec{F} = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$ is conservative over its natural domain and find a potential function for it.
14. Evaluate the line integral

$$\int_{(1,1,1)}^{(2,3,-1)} ydx + xdy + 4dz$$

over any path from $(1, 1, 1)$ to $(2, 3, -1)$.